George William Hill: The Eighth Bruce Medalist

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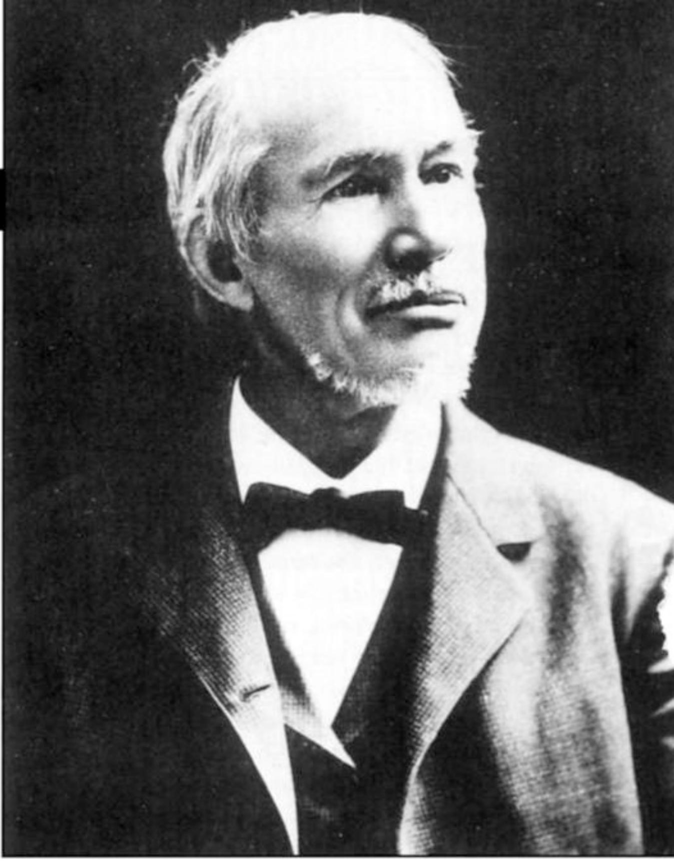

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George William Hill
3 March 1838 — 16 April 1914
1909 Bruce Medalist

A visiting lecturer, welcomed at a formal dinner at the Sorbonne in Paris in about 1911, was astonished to hear, "We honor you for several reasons; first because of your own distinguished accomplishments; second, because you are an American; and third and chiefly because you are a countryman of George William Hill."

The American was embarrassed; he had never heard of Hill. To this day, few people have.

G. W. Hill was a celestial mechanician, a mathematician who developed new and improved methods to compute the motions of the Moon and planets. Celestial mechanics began with Isaac Newton, who declared in the 1680s that all motions in the sky could be obtained from his law of motion and his law of universal gravitation.

The resulting equations are easy to write down for anyone who has learned the calculus which Newton invented (simultaneously with Gottfried W. Leibniz). As every physics and astronomy student learns, if there are only two point masses, then each moves in a perfect, unchanging ellipse with their center of mass at one focus. To most students it is sufficient to treat the Sun and the center of mass of the Earth-Moon system as such a two-body problem, find the motions, and then treat the Earth and Moon as another two-body problem to find their relative motion. Newton, in his *Mathematical Principles of Natural Philosophy* of 1687 already took into account part of the perturbation due to the Sun to compute the Moon's position in the sky.

Over the next two centuries, telescopes grew larger, and astronomers learned to measure positions in the sky with more precision. Navigators demanded tables that would allow them to determine their positions on Earth within one hundred yards. To achieve such precision, theorists had to develop improved methods for computing the orbits of the Moon and planets. Much of the analytical mechanics we use today was developed in the academies of Czars Peter the Great and Catherine the Great in St. Petersburg by the Swiss-born mathematician Leonhard Euler. The theory was further improved by such great mathematicians as Joseph Louis Lagrange, Pierre Simon Laplace, and Karl Friedrich Gauss. It was found that the problem of three bodies, such as the Sun, Earth, and Moon, cannot be solved exactly in terms of formulas into which one could plug initial conditions and get positions for all time to come. Instead, approximations must be made, quantities expanded in powers of small terms,[1] and the series summed with the numbers evaluated to many decimal places.

It is difficult in the computer age to appreciate how hard the problem was. By the late 19th century, observations were so precise that in order to predict adequately the motion of the Moon, it was necessary to take into account the small perturbing effects of all of the other planets and also the fact that the Earth is not a perfect sphere. The most-used theory, developed by Peter Andreas Hansen, the director of a private observatory in what is now Germany, used numbers specific to the Moon and could be applied only to that body. Hansen's tables, published in 1857, gave the position of the Moon to within two arcseconds for the entire century 1750 - 1850. The other method then available, developed by Charles Delaunay at the Paris Observatory in the 1860s, used algebraic symbols, so that, with the appropriate numbers inserted, it could be applied to the Earth's satellite or that of any other planet. Delaunay's method was considered superior by mathematicians, but its creator drowned in an accident in 1872 before he could provide the numbers that were needed by practical astronomers and navigators. The producers of almanacs relied primarily on

Hansen's tables.

Enter G. W. Hill, one of the most modest and retiring individuals ever to receive the Bruce medal.

Hill was born in New York City, but from the age of eight he lived on a farm some twenty-five miles up the Hudson River at West Nyack, where his father was a portrait painter and farmer. At Rutgers College, Hill came under the influence of the mathematics professor Theodore Strong, a friend of Nathaniel Bowditch, the American translator of Laplace's great work on celestial mechanics. Under Strong's tutelage, Hill studied Laplace, Lagrange, and others of the European masters, even going back to Euler, whose fundamental work was no longer read by most mathematicians of the time.

Hill's mastery of mathematics soon became evident. He began publishing his own contributions in 1859, the year he received his bachelor's degree, and two years later, while furthering his education at Harvard, he published a prize-winning article on the constitution of the Earth.

That same year, 1861, Hill joined the staff of the Nautical Almanac Office, then in Cambridge, Massachusetts. Preferring to work undisturbed, he moved back to the family farm in West Nyack, where he performed nearly all of his work. In the 1860s and '70s he made significant improvements in the theory of the motion of the Moon. To avoid computing series with twenty-seven terms, he invented a new method involving an infinite determinant, and obtained the mean motion of the Moon's perigee (the point in its orbit closest to the Earth) to 13 decimal places. Hill's method was the first to be based on periodic solutions closer to the true orbits than the simple Keplerian ellipses.

Hill's work became essential for others working in the field. These included the two who had predicted the existence and position of Neptune before its discovery, Urbain Jean Joseph LeVerrier at the Paris Observatory and John Couch Adams at Cambridge University, and George H. Darwin, son of the biologist. The most important was the great French mathematician Henri Poincaré, who greatly extended Hill's concept of periodic solutions and showed that some of the infinite series used actually diverge.

In 1877 Simon Newcomb became director of the Nautical Almanac Office, which had moved to Washington eleven years earlier. Newcomb soon began his great program to redetermine, both theoretically and observationally, the motions of all of the bodies in the solar system. Taking the theory of the inner and outermost planets for himself, he assigned that of Jupiter and Saturn to Hill, by this time a recognized master of celestial mechanics. The two most massive planets are the most difficult because of their significant effects on each other.

Hill was prevailed upon to move to Washington in order to discuss the work with others and to take advantage of the large corps of computers (these were people, not machines) who did much of the numerical work. Hill much preferred to do his own computing, but Newcomb was in a hurry. It is clear that Hill hated Washington, and he was not exactly fond of the strong-willed Newcomb, although the latter referred to Hill in his autobiography as "the greatest master of mathematical astronomy during the last quarter of the nineteenth century."

After ten years Hill completed the work on Jupiter and Saturn (his work would be used in computing the Nautical Almanac until 1960) and promptly returned to the farm in West Nyack, where he lived among his eight siblings and continued to work on his own. He never married.

In 1898 Hill was persuaded to take a special lectureship at Columbia University endowed by Miss Catherine Wolfe Bruce, and carrying a salary of $1000 per year. According to the letter from the university president to Hill, "This fund has been given to us especially to enable us to give to you a connection with the University, in recognition of the eminent services which you have already rendered to astronomy . . . We shall be glad to have you offer such lectures as you please; but it is understood that no duty which is not welcome to you is involved in the acceptance of this appointment."

Hill prepared two courses in celestial mechanics, but in three years students appeared only for the elementary course, and then only once. That year he journeyed to New York every Saturday to lecture to three graduate students, one of them Frank Schlesinger, who would win his own Bruce medal many years later. Hill laboriously wrote out every step of his lengthy mathematical derivations and gave his notebooks to the University when he resigned in 1901. Over the protests of university officials, he insisted on returning the last year's salary, writing, "Circumstances, over which I could have had very slight control, have prevented me from earning it."

Hill's work was greatly appreciated by the few who could make use of it. Poincaré referred to him as "not only an able artist and a curious searcher, but an original and profound inventor."

For recreation Hill submitted solutions to problems published in mathematical magazines. He took long walks and made two lengthy canoe trips in Canada. The closest thing to a self-indulgence in his life was a large scientific library, collected so that he would not have to leave the farm. ■

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1. The concept of power series is an extremely useful one. For example, it is not difficult to show that the quantity

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots,$$

the actual sum containing an infinite number of terms. If x is quite small, it is sufficient to keep only the first few terms. You can use your calculator to show that

$$\frac{1}{1-.01}$$

equals $1 + .01$ to a good approximation and equals $1 + .01 + .01^2$ to an even better one. If, however, x is greater than or equal to 1, then the method fails, and the series is said to *diverge*.