Sonoma State University Physics 314
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A Relativity Problem

Prob. 1-9 from Tipler, *Modern Physics*, 2nd ed. (Worth, 1978)
Two observers agree to test time dilation. They use identical clocks, and one observer in frame S' moves with speed $v = 0.6c$ relative to the other observer in frame S. When their origins coincide, they start their clocks. They agree to send a signal when their clocks read 60 min and to send a confirmation signal when each receives the other's signal. (*a*) When does the observer in S receive the first signal from the observer in S'? (*b*) When does he receive the confirmation signal? (*c*) Make a table showing the times in S when the observer sent the first signal, received the first signal, and received the confirmation signal. How does this table compare with one constructed by the observer in S'?

Solution:

Start at $x = x' = 0$, $t = t' = 0$. $v = 0.6c$ $\gamma = 5/4$ $\gamma v = .75c$

As measured in frame S, observer in S is always at $x = 0$
observer in S' is always at $x = vt = 0.6ct$

As measured in frame S', observer in S' is always at $x' = 0$
observer in S is always at $x' = -vt' = -0.6ct'$

Observer in S sends his first signal when his clock reads 60 min, i.e., at $x = 0$, $t = 60$ min. Using the Lorentz transformation, this occurs at $t' = \gamma(t - vx/c^2) = \gamma(60 \text{ min} - 0) = 75$ min and at $x' = -0.6ct' = -45$ c-min. From this point on, this signal is at $x = c(t - 60 \text{ min})$ and at $x' = -45\ c\text{-min} + c(t' - 75 \text{ min})$.

Observer in S' sends her first signal when her clock reads 60 min, i.e., at $x' = 0$, $t' = 60$ min. Using the Lorentz transformation, this occurs at $t = \gamma(t' + vx'/c^2) = \gamma(60 \text{ min} - 0) = 75$ min and at $x = 0.6ct = 45$ c-min. From this point on, this signal is at $x' = -c(t' - 60 \text{ min})$ and at $x = 45\ c\text{-min} - c(t - 75 \text{ min})$.

The first signal from S reaches the observer in S' when positions of signal and observer coincide:

$c(t - 60 \text{ min}) = 0.6ct$	$-45\ c\text{-min} + c(t' - 75 \text{ min}) = 0$
$0.4t = 60$ min	$t' = 75 \text{ min} + 45 \text{ min}$
$t = 150$ min	$t' = 120$ min
$x = 0.6ct = 90\ c$-min	$x' = 0$

Note that the travel time of the signal in $S = \gamma$ times the travel time in S'.

The first signal from S' reaches the observer in S when positions of signal and observer coincide:

$45\ c\text{- min} - c(t - 75 \text{ min}) = 0$	$-c(t' - 60 \text{ min}) = -0.6ct'$
$t = 120$ min **answer to (a)**	$t' = 150$ min
$x = 0$	$x' = -0.6ct' = -90\ c$-min

Each observer sends a confirmation signal upon receipt of the first signal. The confirmation signal from the observer in S is at

$x = c(t - 120\text{ min})$ $\qquad\qquad$ $x' = -90\ c\text{-min} + c(t' - 150\text{ min})$

This signal reaches the observer in S' when

$c(t - 120\text{ min}) = 0.6ct$ $\qquad\qquad$ $-90\ c\text{-min} + c(t' - 150\text{ min}) = 0$

$t = 300\text{ min}$ $\qquad\qquad$ $t' = 240\text{ min}$

$x = 0.6ct = 180\ c\text{-min}$ $\qquad\qquad$ $x' = 0$

The confirmation signal from the observer in S' is at

$x = 90\ c\text{-min} - c(t - 150\text{ min})$ $\qquad\qquad$ $x' = -c(t' - 120\text{ min})$

This signal reaches the observer in S when

$90\ c\text{-min} - c(t - 150\text{ min}) = 0$ $\qquad\qquad$ $-c(t' - 120\text{ min}) = -0.6ct'$

$\boxed{t = 240\text{ min}}$ **answer to (b)** $\qquad\qquad$ $t' = 300\text{ min}$

$x = 0$ $\qquad\qquad$ $x' = -0.6ct = -180\ c\text{-min}$

(c)

Event	x (c-min)	t (min)	x' (c-min)	t' (min)
Start	0	0	0	0
S sends first signal	0	60	-45	75
S receives first signal and sends confirmation	0	120	-90	150
S receives confirmation	0	240	-180	300
Start	0	0	0	0
S' sends first signal	45	75	0	60
S' receives first signal and sends confirmation	90	150	0	120
S' receives confirmation	180	300	0	240

Each observer receives a signal every 120 min. If each constructs a table of just what he or she observes, S's table consists of the first two columns of the first four rows of the above, and S''s table consists of the last two columns of the last four rows. The two tables are identical.

Graphical view: